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Solving rank-constrained semidefinite programs in exact arithmetic

Simone Naldi

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Abstract

We consider the problem of minimizing a linear function over an affine section of the cone of positive semidefinite matrices, with the additional constraint that the feasible matrix has prescribed rank. When the rank constraint is active, this is a non-convex optimization problem, otherwise it is a semidefinite program. Both find numerous applications especially in systems control theory and combinatorial optimization, but even in more general contexts such as polynomial optimization or real algebra. While numerical algorithms exist for solving this problem, such as interior-point or Newton-like algorithms, in this paper we propose an approach based on symbolic computation. We design an exact algorithm for solving rank-constrained semidefinite programs, whose complexity is essentially quadratic on natural degree bounds associated to the given optimization problem: for subfamilies of the problem where the size of the feasible matrix is fixed, the complexity is polynomial in the number of variables. The algorithm works under assumptions on the input data: we prove that these assumptions are generically satisfied. We also implement it in Maple and discuss practical experiments.

Keywords. Semidefinite programming, determinantal varieties, linear matrix inequalities, rank constraints, exact algorithms, computer algebra, polynomial optimization, spectrahedra, sums of squares.

1 Introduction

1.1 Problem statement

Let $x = (x_1, \ldots, x_n)$ denote a vector of unknowns. We consider the standard semidefinite programming problem with additional rank constraints, as follows:

$$(\mathrm{SDP})_r \qquad \begin{array}{rl} \inf_{x\in\mathbb{R}^n} & \ell_c(x) \\ s.t. & A(x) \succeq 0 \\ & \mathrm{rank}\, A(x) \le r \end{array} \qquad (1)$$

Here $\ell_c(x) = c^T x$, $c \in \mathbb{Q}^n$, $A(x) = A_0 + x_1 A_1 + \cdots + x_n A_n$ is a symmetric linear matrix with $A_i \in \mathbb{S}_m(\mathbb{Q})$ (the set of symmetric matrices of size m with entries in $\mathbb{Q}$), and r is an integer, $0 \le r \le m$. The formula $A(x) \succeq 0$ means that $A(x)$ is positive semidefinite (*i.e.*, all its eigenvalues are nonnegative) and is called a linear

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